**PHILOSOPHICAL VIEWS OF ELEMENTARY AND SECONDARY MATHEMATICS TEACHERS**

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**Abstract**. This study examines philosophical views of elementary and secondary teachers in different classes based on the Ernest’s classification which covers a wide range of philosophical beliefs about the nature of mathematics. Given that mathematics appears with a different nature at different levels and elementary teachers teach mathematics alongside other subjects while secondary teachers specialize, we want to compare the views of the two groups in each class. Educational implications are also discussed at each class and level.

**Key words:** Constructivism; Falliblism; Formalism; Logicism; Platonism.

**Introduction**

The interaction between mathematics and philosophy has been established for a long time. This interaction is to the extent that according to Frege, “A philosopher who has no connection to geometry is only half a philosopher, and a mathematician who has no philosophical vein is only half a mathematician” (as cited in Aspeitia, 2000, p. 25).From an educational point of view, there is a close and effective relationship between mathematics and philosophy too.This topic is more related to teachers’ beliefs, include different branches which are effective in teaching mathematics, such as belief about the nature of mathematics, teaching mathematics, and learning mathematics (Ernest, 1989). In this regard, other branches and sub-branches have also been suggested by researchers, such as “beliefs about students, beliefs about teachers’ own ability to do mathematics, to teach mathematics, etc” (Liljedahl & Oesterle, 2014, p. 584).

Despite the importance of all the cases, teachers’ belief about the nature of mathematics is emphasized as one of the most important types of beliefs as for mathematics education. Lerman (1983) pointed out that teachers’ beliefs about the nature of mathematics can affect their views about teaching mathematics. Ernest (1989) also mentioned that teachers’ beliefs about the nature of mathematics is basic for their mental models regarding mathematics education.

As a matter of fact, mathematics appears at different levels with different natures (Howson, 1998). In elementary school, mathematical concepts are more tangible and perceptible, so teachers use teaching aids and real-world examples. In secondary level, by entering discussions such as negative numbers and infinity, mathematics gradually becomes semi-tangible and then abstract. According to Howson (1998) teachers should prepare students for these transitions with respect to the nature of mathematics.

In addition to considering the nature of mathematics at the different levels, mathematics is taught different in terms of the number of teachers and their specializations. Commonly, elementary school teachers teach mathematics along with other subjects such as experimental science while at the secondary level, mathematics is taught by specialist teachers. An important issue here is to compare the belief of teachers who teach mathematics along with other subjects in the elementary level with teachers who only teach mathematics in the secondary level. In so doing, we need a theoretical framework to classify and analyze the beliefs of teachers from both levels and examine the relationship between them exactly.

**Theoretical framework**

Regarding the classification of teachers’ beliefs, in this study we use Ernest’s (1985) framework which covers a wide range of philosophical beliefs about the nature of mathematics. According to Ernest (1985), mathematics consists of five philosophical classes as Logicism, Formalism, Constructivism, Platonism and Falliblism. Logicism says that mathematics is a part of logic. From the perspective of Formalism, mathematics is a meaningless game that is played with symbols on paper and following rules. Platonism is a belief that says mathematics has an independent existence and is discovered by human beings. Constructionism discusses that mathematics is made by human, and the important thing is the process of making, not only the final product. Falliblism is the view that mathematics is what mathematicians do, with all the flaws inherent in any human activity or creation.

According to Ernest (1985), the first four mentioned philosophies of mathematics provide only a few concepts as for mathematics education.Ernest discussed thatFalliblism is a view that is more relevant to mathematics education which emphasizes individuals as creators of mathematics. We have the main question in this research: Is there significant difference between the philosophical views of elementary and secondary teachers about the nature of mathematics in different classes of the Ernest's framework? In the process of finding the answer to this main question, we also investigate the most and least beliefs among teachers at two levels.

**Methodology**

180 mathematics teachers, 90 of whom at the primary and 90 at the secondary level, were randomly selected to participate in this research.All of them had at least 10 years of teaching experience and had at least a bachelor’s degree. Data were collected through a questionnaire (Figure 1).

|  |
| --- |
| About mathematics, I think that ...A: It is the same as logic and its rules can be obtained through axioms of logic.B: It is a kind of game with symbols and rules.C: It has an independent existence which is discovered by human.D: It is invented by human beings, its process making is important, not the final product.E: It is the work done by mathematicians with all the flaws inherent in the human activities.F: None of the above items, but … |

**Figure 1.** Questionnaire

The propositions A, B, C, D and E are respectively in line with philosophical views of Logicism, Formalism, Constructivism, Platonism and Falliblism based on the Ernest’s framework. The proposition F was intended for those who thought their views did not match any of the above. To avoid side discussions and the influence of the names of each proposition, the propositions were presented without titles so that the participants do not get involved with unnecessary discussions according to the purpose of this research.

Although there was no time limit to respond, the average time to answer the questions was about 3 minutes. Responding in such a short period of time could indicate that the participants did not spend much time thinking in the moment and they chose what they already believed.

**Results**

Table 1 shows the frequency of answers in the elementary and secondary levels separately.

**Table 1.** The frequency of responses for each item

|  |
| --- |
|  A B C D E F Total  |
| Elementary 44 27 10 3 3 3 90Secondary 46 4 20 5 2 13 90 Total 90 31 30 8 5 16 180  |

According to the aim of this research, we sought to check whether there is a significant difference between different classes of the elementary and secondary teachers’ beliefs

To do so, Fisher’s exact test was used for comparison between each proposition. As Table 2 shows, there was a significant difference in proposition B between the views of the two groups.

**Table 2.** Fisher’s Exact Test (proposition B)

|  |
| --- |
|  Value df Asymptotic Significance Exact Sig. Exact Sig.  (2-sided) (2-sided) (1-sided) |
| Pearson Chi-Square 20.615a 1 .000Continuity Correctionb 18.861 1 .000 Likelihood Ratio 22.698 1 .000 .000 .000 Fisher's Exact TestN of Valid Cases 180 |

|  |
| --- |
|  |
| a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 15.50. |
| b. Computed only for a 2$×$2 table |

Moreover, Fisher’s exact test assured that there is no significant difference between the views of the two groups regarding other propositions.

As for proposition F included 3 elementary and 13 high school mathematics teachers.

Almost all of them provided a mixed view. Meanwhile, most of the answers included both propositions C and D, that is, they considered mathematics to include both inventions and discoveries.

In total, the highest frequency in both elementary and secondary levels is related to proposition A, which considered mathematics to be logic and inference through logical axioms (Table1). Contrary to what Ernest stated and considered the view of Falliblism to be more educationally useful than other views, in this research, the least frequency was related to proposition E.

**Discussion**

Proposition B implies the view of Formalism and , as Table 1 shows, elementary teachers were more in favor of this view than the secondary teachers. These finding is neither in accordance with the nature of mathematics in elementary school nor in accordance with the natural expectation of teaching mathematics along with other subjects in this level. Although mathematics at higher levels has an abstract nature, at the elementary level it appears consistent with intuition. Therefore, it might be expected that elementary teachers who use teaching aids and real-world examples in teaching do not see mathematics as a mere game with symbols. Instead, it was expected that elementary teachers would have imagined the nature of mathematics under the effect of other subjects with an experimental nature.

In the Ernest classification, there are five propositions, but because there was a possibility that it would not convince some of the teachers’ desired options, proposition F was added. In proposition A, the frequency of elementary teachers was 44 and the frequency of secondary teachers was 46; therefore, there was no significant difference. Proposition A introduced mathematics as logic whose rules can be obtained through the axioms of logic. It seems that this view has always had its supporters among teachers. This view was the most popular in both groups in our study.

Proposition C says that mathematics has an independent existence and is discovered by humans. It is true that there is no significant difference in this matter but elementary teachers had a frequency of 10, while the frequency of secondary teachers was 20. This difference can be justified according to the nature of mathematics in high school. This difference is also not very consistent considering the change in the type of concepts from primary to secondary. Maybe if the frequencies were the opposite, it would be more justified. From an educational point of view, teachers should prepare students in the transition from primary to secondary level to face concepts that are different in nature. At this level, students encounter concepts and definitions created by mathematicians. As Vinner (1991) mentioned, ‘Definitions are “man made”. Defining in mathematics is giving a name’ (p. 66), and this is more searchable in secondary to elementary mathematics.

Another important point was that the most common belief among primary and secondary teachers is that mathematics is the science of logic. It is necessary to investigate what positive and negative consequences this view can have concerning mathematics education. For example, one of the negative consequences can be that it becomes difficult to accept seemingly illogical parts, and maybe parts that are contrary to intuition are not considered as mathematics.

The transition from tangible to abstract concepts in different levels may be done better when teachers’ beliefs are also formed in line with the nature of the concepts of that level.This calls for targeted discussions about philosophical beliefs about mathematics among teachers.Therefore, further studies on teachers’ philosophical beliefs and their consequences at different levels from educational viewpoint is recommended.

**Conclusion**

According to our findings, it can be concluded that there is a significant difference between the views of elementary and secondary mathematics teachers about the nature of mathematics related to the view of Formalism and considering mathematics as a game with symbols and rules. Contrary to expectations, considering the nature of the concepts in elementary level and the influence of other subjects, elementary teachers had more Formalism views than the secondary teachers. Moreover, while as Ernest has stated, the best belief that can be effective in mathematics education is Fallibility, we found that among the teachers, this belief has the least frequent.

**References**

Aspeitia, A. A. B. (2000). *Mathematics as Grammar: 'Grammar' in Wittgenstein's Philosophy of Mathematics During the Middle Period*. Dissertation, Indiana University.

Ernest, P (1985). The philosophy of mathematics and mathematics education. *International Journal of Mathematical Education in Science and Technology, 16*(5), 603-612.

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching*, *15*(1), 13-33.

Howson, G. (1998). Mathematics and Common Sense. In Alsina, C., Alvarez, J. M., Hodgson, B., Laborde, C. and Pérez, A. (Eds) 8*th* *International Congress on Mathematical Education. Selected Lectures*, Seville. Spain: S.A.E.M. THALES, 257–269.

Lerman, S. (1983). Problem-solving or knowledge centered: The influence of philosophy on mathematics teaching. *International Journal of Mathematical Education in Science and Technology, 14*(1), 59-66.

Liljedahl, P., & Oesterlei, S. (2014). Teacher beliefs, attitudes, and self-efficacy in mathematics education. In S. Lerman (Ed.), Encyclopedia of mathematics education (pp. 583–586). Dordrecht, The Netherlands: Springer.

Vinner, S. (1991). The role of definitions in the teaching and learning of mathematics. In D. Tall

(Ed.), *Advanced mathematical thinking* (pp. 65–80). Dordrecht: Kluwer Academic

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